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RESOURCES: A RISK BASED APPROACH

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ABSTRACT

This article describes a theoretical framework and provides an illustrative case study for optimal internal audit resource allocation using a risk approach. A risk index is derived for the audit unit portfolio of a particular company based on an additive model and using pairwise comparisons to derive weighings for risk factors. Tests for intra- and inter-judge consistency of the pairwise comparisons are developed. An integer programming algorithm which employs the risk index is then developed to guide optimal resource allocation within the internal audit department. It is demonstrated that, given diminishing marginal risk reduction, the resource allocation problem may also be solved using simple marginal analysis.

OPTIMAL ALLOCATION OF INTERNAL AUDIT RESOURCES : A RISK BASED APPROACH

BACKGROUND

The objective of internal audit is to assist members of the organization in the effective discharge of their responsibilities [The Institute of Internal Auditors, 1981, p. 1]. In that sense internal audit helps to maximize the value of the firm, or to minimize losses from intentional or unintentional actions that harm the firm, see for example [Barrett, 1984; Mroch, 1987; Saywer, 1981; Miltz and Willekens, 1990].

In order to use internal audit resources most effectively, it is therefore necessary to evaluate the potential losses that face the firm. Boritz [1983] uses the term "Audit-Portfolio Management" within the broader concept of developing an overall audit planning. It is convenient to divide the firm into a set of audit units. The total expected loss for the firm is then the sum of the expected losses in each audit unit. The expected loss in an audit unit will depend on that unit's specific characteristics, termed audit risk factors.

The first task must therefore be to estimate audit risk associated with each audit unit. Next, the internal auditor faces the challenge of constructing a resource allocation algorithm to optimally reduce the assessed risk.

Throughout the paper reference is made to a case study of ABC Inc., a disguised European based transnational corporation. ABC's internal audit department provides worldwide internal audit coverage for 57 affiliates.

IDENTIFICATION OF AUDIT UNITS AND RISK FACTORS

To compute a risk ranking for the audit units the scheme in exhibit 1 is proposed, the logic of which is followed in the text.

Patton, Evans and Barry [1983] describes the use of an additive model for the purpose of internal audit risk analysis. The general concept is that the total risk in an audit unit equals the sum of risks related to N risk factors F1, F2, ... FN. Mathematically, for each audit unit i, the risk Ri is given by :

$$Ri = \sum_{j=1}^{N} Wj \times Fij,$$

where factor Fj has importance Wj and value Fij for unit i. The weights Wj which can be viewed as constant coefficients of the model and the values Fij which are variable across units have to be derived later.

Chambers [1981] suggests a multiplicative model, where the general concept is that total risk Ri in an audit unit equals the product of the probability Pi of a loss in the unit with the amount Ai of potential loss. Mathematically, for each audit unit i, the risk Ri is given by :

$$Ri = Pi \times Ai.$$

The former additive model predominates internal auditing literature, see for example the review by Selim [1987]. It provides a solid theoretical basis for risk assessment and will be used in this paper in preference to the multiplicative approach, which requires direct estimates of the probability of loss at the unit level.

Harold [1989] asserts correctly the need to clearly define what constitutes an auditable unit. For ABC Inc. audit units were classified into 4 categories A, B, C and D, containing a total of 57 units (step 3 in exhibit 1). Categories varied into degree of interaction with and type of reporting to ABC's headquarters. The overall criterion used was to select those units who's sales affected ABC's worldwide sales significantly.

A major element of risk assessment is the identification of those factors that influence the riskiness of an audit unit. Previous research in this area, see for example [Patton, Evans and Barry, 1983; Noxon, 1980; Siers and Blyskal, 1987], provides a list of relevant risk factors, which can serve as a baseline to specify the risk factor construct in any particular organization.

For ABC Inc., a delphi approach was used to gather information and reach consensus on the selection of risk factors (step 4 in exhibit 1). Five internal audit experts within the company were asked to specify which risk factors contributed to internal audit risk. The experts' input was listed and the various factors were grouped. This grouped list was redistributed to all experts for possible modifications. Input from the panel was collected a second time, which resulted in six different groups of risk determinants. Definitions for these six groups of risk determinants were then discussed within the panel. All experts agreed with the final specification, and so each group was associated with a generic risk factor[1]. The results of this exercise were in line with the earlier mentioned literature and are summarized in exhibit 2.

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1 While there is no theoretical reason to restrict the number of factors, there are some behavioural reasons to do so. Psychological research indicates that the maximum number of factors for which decision makers can make meaningful judgements will vary from five to nine (see for example [Miller, 1956] and [Saaty, 1977]).

COMPUTATION OF RISK FACTOR WEIGHTS AND THE RISK INDEX

Next, the model requires an assessment of the relative contribution of each selected risk factor to the unit's risk. Therefore it is necessary to assign a weight to each of the risk factors. Weights may be computed either directly or indirectly. In the direct estimation approach, each expert directly estimates the importance for each of the N risk factors on a percentage scale[2]. Assuming K experts, K sets of N weights, **Wk**=(Wik),i=1..N, are obtained. Each Wik has a value between 1 and 100, and for each expert k, the sum of the Wik should add to 100 %.

Alternatively, Siers and Blyskal [1987] suggest the use of conjoint analysis for the estimation of risk factor weights. A panel of K experts is asked to rank preference cards according to risk. Each preference card describes a fictitious unit with specific levels of risk for a selected number of risk factors. Using multiple linear regression and the input of K experts, a weight is derived for each possible level of each risk factor. This results in a slight modification of the risk assessment model. The risk Ri in unit i is then given by :

$$Ri = \sum_{j=1}^{N} Wj(Fij) \times Fij,$$

where factor Fj has value Fij for unit i and importance Wj depending on the level factor Fj takes.

Further Patton, Evans and Barry [1983] describe the pairwise comparison method, based on scaling methods introduced in [Saaty, 1977]. A panel of K experts compares, pairwise, the N risk factors. K comparison matrices **Ck**=(Cijk),i=1..N,j=1..N are obtained. The element Cijk indicates the relative importance of

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2 This approach is in [Boritz, 1983] and [Patton et al., 1983].

factor Fi over factor Fj on a scale with P values X1 to XP, as estimated by the kth expert[3]. It can then be shown that for each matrix **Ck** the eigenvector associated with the largest eigenvalue is an estimate of expert k for the set of N weights **Wk**.

The method of direct estimation is quick when differences in importance are clear. When a fine judgement is needed, David [1988] points out the advantage of the pairwise comparison method which reduces the effect of extraneous influences from the presence of other factors. Siers' approach is more complex than the pairwise comparison method from a computational point of view. The design of preference cards used in gathering expert opinions, makes input more incomplete compared to the pairwise comparison method. From this reason we think the pairwise comparison method is superior to other methods in deriving importance weights for risk factors[4].

In order to derive a comparison matrix, each expert k is required to compare pairwise, the N risk factors using a comparison scale X1..XP. The expert assigns a score Cijk from the scale X1..XP to each pair (Fi,Fj) to indicate his estimate of the relative importance of factor Fi to factor Fj. This need only be completed for the upper half of the comparison matrix **Ck**, since the score for (Fj,Fi) can be assumed to be the reciprocal of the score (Fi,Fj).

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3 The choice of the comparison scale is discussed by Saaty [1977]. A nine-point scale (P=9) is suggested as giving the best results with respect to the computation of the underlying importance weights. This scale is in exhibit 3. If in the comparison (Fi,Fj) Fi is less important than Fj, the reciprocal of a scale value is entered for Cijk. Diagonal elements in each **Ck** matrix are equal to one.

4 David [1988] provides a complete treatment of the subject of pairwise comparison, but his work is limited to the consideration of preference judgements.

The derivation of the vector of weights **Wk** from the matrix **Ck** for each expert k is described in detail in Saaty [1977] and briefly summarized in this paragraph. We will drop the index k for clarity. Suppose the vector of importance weights **W**=(Wi),i=1..N is given. If one should compare then the N factors in pairs according to their relative weights, one obtains a matrix **A**=(Aij),i=1..N,j=1..N, with :

$$Aij = Wi / Wj.$$

Postmultiplying **A** by **W**, we obtain :

$$\mathbf{A}\ \mathbf{W} = N\ \mathbf{W}.$$

If then only **A** would be known, and not **W**, we could solve **W** from the above. We obtain :

$$(\mathbf{A} - N\ \mathbf{I})\ \mathbf{W} = 0,$$

which has a nonzero solution if and only if N is an eigenvalue of the matrix **A**. Now all rows of **A** are linear dependent, thus the rank of **A** is one. Hence only one eigenvalue is different from zero. Call this eigenvalue Lmax. In addition, the sum of all eigenvectors equals the trace tr(**A**) of the matrix and

$$tr(\mathbf{A}) = N,$$

so it follows that :

$$Lmax = N.$$

The eigenvector of **A** associated with the eigenvalue Lmax is then a solution for **W**. To compute the eigenvector for a real numbered symmetric matrix, several algorithms exist, see for example [Press et al., 1986] In this theoretical problem where each entry Aij exactly equals Wi/Wj, the eigenvector solution **W** equals any column from **A**. Scaling **W** so that its elements sum to unity gives a unique solution.

Effect of Intra-judge Inconsistency

In practice, estimation of the relative importance of risk factors results in a pairwise comparison matrix **C**=(Cij),i=1..N,j=1..N where each Cij reflects the expert's judgement of the ratio Wi/Wj. Due to inconsistency in the expert's judgement, it is observed in general that :

$$Cij \neq 1 \,/\, Cji,$$

and :

$$Cij \neq Cik \times Ckj.$$

The reciprocity problem can be eliminated if experts are asked to fill in only the upper half of the comparison matrix **C**. Intra-judge inconsistency therefore amounts to non-adherence to the transitivity problem. Saaty [1977] proves that the matrix **C** is consistent if and only if the largest eigenvalue Lmax of **C** equals N[5]. If the matrix **C** is not consistent, then Lmax>N. Deviation from consistency can be measured by the statistic M, where :

$$M = (Lmax - N) \,/\, N - 1).$$

Saaty shows that in the case of small deviations from consistency, the eigenvector **W** corresponding with the largest eigenvalue Lmax of **C** provides an estimate of the underlying

5 Note that when the comparison matrix is computed from the known **W** consistency is always ensured.

vector of importance weights. To test for consistency it is assumed that :

$$Cij = (1 + Dij) \times Wi / Wj.$$

Under the assumption that the Dij are normally distributed on the interval [-1,+1], the null hypothesis H0 that **C** is consistent should be rejected if M>2. Although this test can be useful to obtain a quick view on the consistency of **C**, the authors prefer to develop some alternative tests that do not make any assumptions a priori on the distribution of M.

It is obvious that if a comparison matrix **C** was composed of entries randomly chosen from the comparison scale, the resultant matrix would not be consistent. Therefore, the M statistic computed for a matrix **C** which is expected to be consistent could be compared with the distribution of that statistic obtained from a large sample of matrices with random entries (which can be considered as generally inconsistent). Under the assumption that **C** is consistent, it is expected that the M statistic would lie in the left tail of the derived distribution, that is below a certain cutoff value $M\alpha$, depending on the confidence level α. Appendix A contains tables with cutoff values for different confidence levels and for varying N (number of risk factors) and P (number of points on the comparison scale).

An alternative test for consistency is based on the cell entries of **C** only. Consistency requires that :

$$Cij = Cik \times Ckj, \text{ for all } i,j,k.$$

Hence, for a particular cell entry Cij we can compute N-2 alternatives CikxCkj (we take $k \neq i,j$).

The average alternative :

$$Aij = \left(\sum_{k=1}^{N} Cik \times Ckj \right) / N-2,$$

can then be compared for each (i,j). Note that since we force reciprocity, only the upper half of the matrix **C** has to be considered. One obtains then Nx(N-1)/2 pairs (Cij,Aij). The null hypothesis that Cij equals Aij can then be tested using the nonparametric Wilcoxon MPSR test. For a description of this test, see for example [Siegel, 1956].

For ABC Inc. a panel of five experts is asked to estimate the relative importance on the six risk factors described in exhibit 2. In the comparison a nine-point scale was used as described in exhibit 3. Results and computations for each expert are presented in exhibit 4.

It can be observed that for all experts both tests of consistency revealed in satisfactory results, in that the hypothesis of consistent comparison matrices cannot be rejected at the $\alpha = 0.05$ level for all experts. However the set of N weights varies across experts. This consensus issue is considered below.

Effect of Inter-Judge Inconsistency

For each expert k in the panel of K experts, one can compute the set of importance weights **Wk** from the expert's comparison matrix **Ck** as described above. Each set **Wk** can be accepted if no intra-judge inconsistency in the matrix **Ck** is detected.

Even when each individual expert is consistent in his/her assessment of factor weights, inconsistencies may and will appear across experts. A measure for this lack of consensus, together with a method to derive a consensus solution is proposed.

In general one observes different estimates across experts, or :

$$Wik \neq Wil \text{ if } l \neq k.$$

An indication for the lack of consensus between two experts k and l may be obtained from the correlation coefficient of **Wk** with **Wl**, denoted ρkl. This measure is termed pairwise correlational consensus by Ashton [1985]. Computation of ρkl for each pair of experts results in the correlation matrix **R**, which then measures the consensus across experts. For consensus, one expects entries in **R** to be positive and significantly different from zero.

Einhorn [1974] however, points to the fact that experts may differ in opinion due to their different experience and training. This is not an error but reflects each expert's way of organizing information. In the case of multiple measurements (traits) each estimated by the different experts (judges), one could look at the multitrait - multijudge correlation matrix, and examine patterns within this matrix. In the case of risk factor weight estimation one deals with only one measurement per expert, so the multitrait - multijudge matrix boils down to Ashton's pairwise correlation matrix.

The pairwise correlation matrix for ABC Inc. is presented in exhibit 5. Not all correlations are positive, which indicate a lack of consensus amongst experts. If the pattern in the correlation matrix is examined more closely, it may be observed that experts 1, 2 and 4 agree, and form a cluster, whereas expert 3 and expert 5 each form two further distinct clusters. None of the correlations are significantly different from zero because of the low number of degrees of freedom (N-2).

Different approaches exist for combining each expert's risk factor weight assessment so as to obtain a consensus solution.

David [1988] and Barzilai, Cook and Kress [1986] use a network model to represent ordinal preferences. They show that the consensus formation problem can be modelled as a generalized network. For risk factor weighing however this approach cannot be used since comparisons are based on a nine-point preference scale. What is crucial in this approach, however, is that consensus is reached on the pairwise comparison data level (see exhibit 4), and not on the derived weighings level. In that way one uses all available information and circumvents the lack of positive correlation as described in the paragraph above.

The first approach used here is a to obtain a central tendency per cell entry in the pairwise comparison matrix such as the mean or the median, and then compute factor weights from that newly derived matrix. The factor weights obtained can then be discussed by all experts. If no consensus is reached, one can go back to the 'central tendency' comparison matrix, and attempt to obtain consensus on each entry. If the corresponding vector of weights is then recomputed, it necessarily presents the panel's consensus solution. The central tendency comparison matrix for ABC Inc. was computed by taking median values per cell entry (see exhibit 6). Note that the consensus solution by accident coincides with the weights obtained for expert 1 (which happened to be the most experienced judge in the panel).

An alternative method to derive a consensus solution is based on [Caroll and Chang, 1970] from the theory of three-way multidimensional scaling problems. If consensus for a panel of experts holds, we can assume that each expert's weights **Wk**=(Wik),i=1..N are a linear function of a common set of weights **Z**=(Zi),i=1..N :

$$Wik = Vk \times Zi.$$

The problem is then, given the computed sets **Wk**, to find **V**=(Vk),k=1..K and **Z** so that :

$$S = \sum_{i=1}^{N} \sum_{k=1}^{K} (Wik - Vk \times Zi)^2,$$

is minimal.

Denoting **W**=(Wik),i=1..N,k=1..K, the first equation transforms into :

$$^t\mathbf{W} = \mathbf{V}\ \mathbf{Z}.$$

Suppose an initial estimate of **Z**, say **Z0** is given, S is minimized given **Z0** if **V** is the least squares solution of

$$^t\mathbf{Z0}\ \mathbf{V} = \mathbf{W}.$$

Call this solution **V1**. The vector **Z** can then be recomputed from

$$\mathbf{Z} = {}^t\mathbf{W}\ \mathbf{V1}^{-1}.$$

This iteration can be repeated until the stress function S is minimized.

The results obtained by the Caroll method for ABC Inc. are presented in exhibit 7. When compared to the solution derived from the central tendency comparison matrix in exhibit 6, it can be observed that both methods do not result in identical solutions. Both solutions however, are very similar and highly correlated (correlation of 0.959).

Computation of Risk Index

Having computed a consensus solution for the risk factor weights, each unit's particular risk score can be computed by filling in the unit specific risk factor values in the model. A direct estimation or pairwise comparison method may be used to derive these units specific risk factor values. The pairwise comparison method results in the comparison of pairs of units per factor. For ABC Inc. this would require a total of 9576 comparisons to be made. To avoid this exercise, and because of the preponderance of objective data[6] the direct estimation is preferable here. Direct estimation requires an assessment of the magnitude of each of the risk factors in every unit. Each risk factor score is typically based on a five-point risk scale ranging from 1 (low risk) to 5 (high risk) with 2, 3 and 4 as intermediate values.

For ABC Inc. the extent of risk for each of the six factors was assessed for each of the 57 audit units. Results are presented in exhibit 8. The use of a mixture of objective and subjective data is again stressed. Risk assessment based on these objective criterion accounted for approximately 60 % of the total risk assessed for each unit. The remaining 40 % of the risk is based on subjective criteria. This subjective risk was estimated by the internal audit director.

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6 Examples of objective data included : total sales, number of employees, previous audit reports, announcements of changes in management, occurrence of new product introductions and presence of activities such as R&D or production.

The proposed additive risk model computes the total audit risk per unit as the weighted sum of the riskiness per factor. Units can then be ranked according to decreasing risk. The risk index for ABC Inc. is also presented in exhibit 8. Risk scores range from 4.44 (for audit unit A15) down to 1.68 (for audit unit A2).

INTERNAL AUDIT RESOURCE ALLOCATION : THE MODEL

In general, auditing literature in the area of resource allocation and audit scheduling formulates the objective of the internal audit in terms of minimizing losses which would occur in the absence of auditing[7]. In risk based auditing, the extent of potential loss related to a particular audit unit is related to the risk index assessed. The more a unit is likely to generate losses if remained unaudited, the higher its derived risk score. The objective of the audit is to reduce the overall risk of loss. Overall loss for the corporation is composed out of the losses per unit. The allocation model developed here, defines the benefit resulting from the audit as the total risk reduction obtained after audit. It is assumed that the assessed risk score per audit unit is proportional to the potential loss generated per unit in the absence of audit. If this were not the case it would be necessary to transform the per unit risk into some other measure which is proportional to expected monetary loss.

The overall risk reduction problem can be represented by an integer programming formulation with the objective function to maximize :

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[7] For example : Wilson and Ranson [1971] view the audit as a loss control mechanism, Patton et al. [1983] minimize overall monetary loss, and, Boritz and Broca [1986] minimize total relevant costs which are defined as expected losses and audit costs over the planning period.

$$RR = \sum_{i=1}^{M} RRi \times Xi, \qquad (1)$$

where RR is the total risk reduction at corporate level assuming M audit units, RRi is the risk reduction obtained for audit unit i, and the dummy variable Xi indicates whether unit i is audited (Xi=1) or not audited (Xi=0).

The cost of the internal auditing activity includes the costs of salaries and overheads of maintaining the internal audit department and the opportunity cost of reducing time spent on other activities. Only the audit costs which can be directly attributed to the audit units (which can be thought of as cost centres), that is only a part of the salary cost of the internal audit department, are relevant costs in our resource allocation model. For planning purposes, the salary cost of an internal audit department over the planning horizon (for example one year) can be defined in terms of the total amount of audit man-hours available, times the standard salary cost per man-hour. The total amount of man-hours available or the total audit time, is limited and needs to be allocated such that benefits of the audit are maximized. However, only that part of this total available audit time which can be allocated to the audit units on a discretionary basis, will be the relevant audit time for the budget constraint in this model[8]. This constraint can be formulated as :

$$\sum_{i=1}^{M} Ti \times Xi \leq T, \qquad (2)$$

[8] Discretionary audit time can differ from total audit time available. For example, special assignments may be imposed by top management, or legal requirements, or internal training and administration may all absorb a certain amount of the available audit time.

where Ti is the time necessary to audit unit i (in man-hours) and T is the total discretionary audit time available over the planning horizon (in man-hours).

The objective function is assumed to take discrete values RRi(j) at well defined levels j of time allocation, Ti(j) for each audit unit i. Thus, both possible risk reduction and necessary audit time is unit and level specific, and audit time can only be allocated to one level of time Ti(j) per audit unit i. Also, the audit time allocated to a unit i cannot take some intermediate value between Ti(j) and Ti(j+1), such that the risk reduction per unit cannot take a value between RRi(j) and RRi(j+1). In this the model differs from the traditional resource allocation models in for example [Patton, Evans and Barry, 1983], which implicitly assume continuous time and benefit functions for each audit unit. The rationale for the approach taken here is that audit procedures and jobs require a certain budgeted amount of time to be completed and to yield a benefit (i.e. risk reduction).

Therefore, before allocating discretionary audit time to the audit units, the total feasible audit work in an audit unit needs to be divided into levels j, ranging from the most basic level of audit (j=1) to the highest level of audit (j=J), with each successive level of audit demanding a well defined amount of additional time ΔTi(j), thus :

$$Ti(j+1) = Ti(j) + \Delta Ti(j),$$

where Ti(j) is the time necessary to audit unit i at level j.

The higher the level of audit time allocated to a unit, the more risk will be reduced, but the amount of risk reduced is unit and level specific. Thus :

$$RRi(j+1) = RRi(j) + \Delta RRi(j),$$

where $RRi(j)$ is the risk reduction resulting from the time allocation to audit unit i at level j. Each additional level presents an extension of the work done at the previous level.

The concept of levels as defined here gives rise to an additional constraint indicating that only one level per unit can be allocated. The problem formulation then becomes :

Maximize :

$$RR = \sum_{i=1}^{M} \sum_{j=1}^{J} RRi(j) \times Xi(j), \qquad (1')$$

subject to :

$$\sum_{i=1}^{M} \sum_{j=1}^{J} Ti(j) \times Xi(j) \leq T, \qquad (2')$$

$$\sum_{j=1}^{J} Xi(j) \leq 1, \text{ for } i=1 \text{ to } M, \qquad (3)$$

$$Xi(j) \in \{0,1\}, \text{ for } i=1 \text{ to } M \text{ and } j=1 \text{ to } J. \qquad (4)$$

A solution for the problem can easily be found using integer programming. The model can be extended by additional constraints. For example, if there is a requirement to reduce risk in a specific audit unit i, to its base level, the following constraint would be included :

$$Xi(J)=1.$$

Patton, Evans and Barry [1983] assume the presence of diminishing marginal returns to additional auditing in any single unit. As more internal audit time is allocated to a unit, the additional benefit of allocating more time decreases, because the more productive audit activities are normally performed first.

In the case of diminishing marginal returns, a solution can be obtained by marginal analysis. For each audit unit i, we consider the defined discrete function of risk reduction per unit of time allocated RRi(j)/Ti(j). Its first forward difference ΔRRi(j)/ΔTi(j) is defined by :

$$\Delta RRi(j)/\Delta Ti(j) = \frac{RRi(j+1) - RRi(j)}{Ti(j+1) - Ti(j)}.$$

For each audit unit i, the RRi(j)/Ti(j)-function is called concave if its first difference is non-increasing, that is :

$$\Delta RRi(j)/\Delta Ti(j) > \Delta RRi(j+1)/\Delta Ti(j+1).$$

RRi(j)/Ti(j) is the benefit from auditing audit unit i at a level j, per unit of time allocated. ΔRRi(j)/ΔTi(j) is the marginal benefit of next level of audit in unit i. A function satisfying the above equation is said to exhibit decreasing marginal return. The selection of each higher level of audit work in a unit is only possible if each preceding lower level is executed. Decreasing marginal return implicitly fulfils this condition.

An optimal audit time allocation can be obtained by the following iteration procedure. A feasible allocation $\hat{T}$ of the total available audit time over the levels of each audit unit can be formulated as :

$$\hat{T} = (T1(J1), T2(J2), \ldots, Ti(Ji), \ldots, TM(JM)).$$

Each iteration results in another feasible time allocation[9]. This procedure is continued until the optimal allocation is found. To start the iteration, a slack audit unit 0 is introduced and each feasible time allocation is now defined as :

$$\hat{T} = (T0, T1(J1), \ldots, TM(JM)).$$

The marginal analysis is then initiated with the feasible allocation $\hat{T}=(T,0,0,\ldots,0)$, thus all available audit time T is allocated to the slack audit unit 0 and for all audit units Ji is set to zero, which implies Ti(Ji)=0. Marginal analysis identifies the unit in which it is the most profitable to increase the allocation of time by one level ($j \rightarrow j+1$), Time from the slack unit 0 is then re-allocated to this unit. Thus per iteration the following trading rule is employed.

1. Let i1 indicate the unit which offers the maximum marginal increase in risk reduction per marginal time increase at the given levels of auditing J1, ..., JM so that :

$$I(\hat{T}) = \frac{\Delta RRi1(Ji1+1)}{\Delta Ti1(Ji1+1)} = \max_{\substack{i=1..M \\ Ji \neq J}} \frac{\Delta RRi(Ji+1)}{\Delta Ti(Ji+1)}$$

2. Audit time is re-allocated by increasing the audit level from $j \rightarrow j+1$ in the audit unit i1 where $I(\hat{T})$ is found, and decreasing the time allocated to the slack audit unit 0.

The trading rule is repeated until T0 equals 0.

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9 $\hat{T} = (T1, T2, \ldots, TM)$, where Ti indicates the total time allocated to unit i for each of the M units is a function of the levels of audit Ji allocated to each unit i. Hence :

$$\hat{T} = \hat{T}(J1, J2, \ldots, JM) = \hat{T}(T1(J1), T2(J2), \ldots, TM(JM)).$$

The model presented here considers both the efficiency and effectiveness of the audit work performed on all audit units[10]. Thus, the possible risk reduction depends not solely upon the assessed risk index as suggested in [Patton, Evans and Barry, 1983], but also upon a unit's responsiveness to auditing, as reflected in the variations in marginal risk reduction across unit and between levels of audit. The identification of audit levels is directly related to the cost effectiveness criterion, and makes the model more attractive for application in practice.

APPLICATION OF THE ALLOCATION MODEL TO ABC INC.

The resource allocation model developed in the previous section was applied to ABC Inc. and an optimal solution for the internal audit resource allocation problem over one year was obtained by both integer programming and marginal analysis.

The internal audit department of ABC Inc. defined three levels of audit. Audit level one is a businessman review, which implies a global assessment of the unit's internal control system which is based on interviews with management without detailed testing. Level two is a detailed review of specified selected cycles, where level three is a complete review of all cycles. Exhibit 9 contains the data necessary for estimating the Ti(j) and RRi(j) values. Clearly the total feasible risk reduction for a unit RRi will be dependent on the risk assessed for that unit TRi. In

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10 Anderson and Young [1988] make the point that previous planning models do not isolate the effectiveness of an audit as a function of the amount of audit work and the actions of the auditee. However, the approach the authors take differs from the one taken here, as they developed a game theoretic model of internal audit resource allocation, which explicitly considers the impact of the auditee's anticipation of the effectiveness of audit work and focuses on possible actions of misappropriations of assets by the auditee.

addition, TRi should be adjusted to reflect a minimal level of inherent risk MRi unavoidable for each unit.

The unit's complexity will influence the standard level of time necessary to perform audit work. Morehead and Myers [1980] use a complexity time diagram based on historical data to compute an audit's time requirements in function of its complexity. In general, the higher the complexity of a unit's operations, the more time it will take to perform a given set of audit activities.

Familiarity of the auditor with the auditee will positively influence both the level of time and the level of risk reduction. These factors will thus influence the efficiency of the audit work, measured in terms of time requirements, and the effectiveness of the audit work measured in terms of risk reduction. The detail of computations and underlying formulas supporting exhibit 10 are in appendix B.

Integer Programming Solution

Time and risk reduction data on ABC Inc. were obtained and used as input to the resource allocation model. The time constraint was set on 6,400 man-hours (reflecting 200 discretionary audit days per year per auditor, 8 hours a day, and a staff of 4 auditors).

The integer program algorithm was solved using a widely available package (LINDO). Exhibit 10 displays the optimal allocation solution over the coming period of one year. A total risk reduction of 43.09 is obtained. Out of the 57 audit units, 44 will be subject to an audit. In none of the units all cycles will be reviewed (third level audit), 22 units will be subject to a review of selected cycles (second level) and 22 units to a businessman review (first level).

Marginal Analysis Solution

The results of the marginal analysis for ABC Inc. are the same as in exhibit 10 (integer programming solution). Given concavity of the RRi(j)/Ti(j)-function, marginal analysis is readily performed by means of a spreadsheet, ranking the audit units and levels by descending ΔRRi(j)/ΔTi(j) values.

CONCLUDING COMMENTS

The risk based resource allocation model outlined here is easily applied; particularly in the presence of diminishing marginal risk returns to audit effort, where the optimal allocation of resources may be achieved using marginal analysis on any commercially available spreadsheet programme. The model is essentially composed of two exercises; the first requiring the generation of an audit unit risk index (by way of pairwise comparisons) and the second being the achievement of maximal reduction of aggregate corporate risk from given audit resources.

In this study, inter-judge concensus was obtained using both a central tendency and a three-way multidimensional scaling approach. The comparison of inter-judge risk factor weightings in the pairwise comparison exercise may well reveal clusters of opinions. It could be envisaged that a broader composition of experts, as compared to this study, might reveal clusters identifiable with different divisions or disciplines within the company.

The marginal worth, in terms of risk reduction, of changes to internal audit resources demonstrates the adequacy of staffing levels. This observation assumes that management can provide a measure of the aggregate acceptable risk level, or that a usable transformation between the risk index and monetary loss is available. In this study the adequacy of staffing levels for meeting management specified risk reduction targets, was not investigated because of the difficulties of obtaining these required measures. In the presence of clusters of opinions on divisional or departemental lines, and given an acceptable aggregate corporate risk measure, it would be particularly interesting to investigate the implication of such clusters for staff allocation and for staffing levels in the internal audit department.

The output of the model provided useful planning information that focussed the resource allocation task in the company investigated. The usefulness of this approach could be enhanced by reprogramming the allocation problem in a dynamic context. Resource scheduling, in addition to resource allocation, could then be considered in a model where audit unit risk is a function of time since last audit (a paper investigating these issues is in progress). It should be noted that the analysis presented here assumes each audit unit to be positioned at maximum risk.

APPENDIX A : DISTRIBUTION OF SAATY'S M STATISTIC TO TEST FOR INTRA-JUDGE CONSISTENCY

	α = .01	α = .05	α = .10	α = .25	α = .50
N = 3					
P = 3	0.0000	0.0000	0.0000	0.0091	0.0678
P = 5	0.0000	0.0000	0.0028	0.0268	0.1087
P = 7	0.0000	0.0013	0.0046	0.0401	0.1527
P = 9	0.0000	0.0018	0.0091	0.0539	0.2178
N = 4					
P = 3	0.0069	0.0202	0.0393	0.0618	0.1261
P = 5	0.0153	0.0477	0.0781	0.1589	0.3007
P = 7	0.0202	0.0709	0.1190	0.2411	0.4854
P = 9	0.0362	0.1003	0.1597	0.3300	0.6765
N = 5					
P = 3	0.0292	0.0543	0.0738	0.1119	0.1732
P = 5	0.0692	0.1320	0.1760	0.2813	0.4271
P = 7	0.1194	0.2196	0.2916	0.4621	0.7204
P = 9	0.1580	0.2978	0.3955	0.6500	1.0295
N = 6					
P = 3	0.0618	0.0914	0.1119	0.1524	0.2047
P = 5	0.1529	0.2269	0.2771	0.3706	0.5006
P = 7	0.2294	0.3698	0.4575	0.6291	0.8352
P = 9	0.3128	0.4963	0.6298	0.8819	1.1880
N = 7					
P = 3	0.0913	0.1245	0.1433	0.1777	0.2218
P = 5	0.2199	0.3030	0.3522	0.4410	0.5442
P = 7	0.3453	0.5074	0.5878	0.7311	0.8927
P = 9	0.5051	0.7126	0.8264	1.0309	1.2686
N = 8					
P = 3	0.1161	0.1462	0.1637	0.1962	0.2329
P = 5	0.2818	0.3605	0.4032	0.4804	0.5677
P = 7	0.4621	0.5929	0.6765	0.7944	0.9339
P = 9	0.6651	0.8588	0.9594	1.1453	1.3290
N = 9					
P = 3	0.1336	0.1626	0.1795	0.2070	0.2403
P = 5	0.3465	0.4089	0.4476	0.5128	0.5886
P = 7	0.5560	0.6748	0.7408	0.8507	0.9673
P = 9	0.7841	0.9690	1.0582	1.2075	1.3730

This table was computed with Monte Carlo simulation (5000 runs) on a IBM 3090/28S mainframe computer using VS/APL software.

APPENDIX B : DETAILED COMPUTIONS FOR RISK REDUCTION AND TIME REQUIREMENTS FOR ABC INC.

Time requirements and risk reduction can be expressed as :

$$RRi(j) = RRi(j, TRi-MRi, Fi),$$

and :

$$Ti(j) = Ti(j, Ci, Fi),$$

where j indicates the level of audit work, TRi the total risk in unit i as computed from the risk model, MRi the minimal risk, inherent to unit i that cannot be reduced, Ci the complexity of the unit and Fi the familiarity with the unit. Ci and Fi can be expressed on a five-point scale.

More specific for ABC Inc., risk reduction RRi(j) for unit i and audit level j is computed as :

$$RRi(j) = A(j) \times (1 - 0.05 \times (5 - Fi)) \times (TRi - MRi).$$

TRi-MRi is the basic risk reduction factor modified for the level of auditing A(j) and familiarity Fi. A(j) is the risk reduction rate equal to 0.4, 0.7 or 1.0 for j equal to 1, 2 or 3 respectively, expressing that subsequent audit levels reduce 40 %, 70 % and 100 % of the basic risk reduction value. Each unit of familiarity increases risk reduction with 50 %.

For ABC Inc., the time Ti(j) to perform audit level j in unit i is computed as :

$$Ti(j) = A(j) \times (1+0.1 \times (5-Fi)) \times (1+0.5 \times (Ci-1)),$$

with $A(j)$ equal to 32, 80 or 200 manhours for j equal to 1, 2 or 3 respectively. This basic time value $A(j)$ to perform audit work at level j independent of a unit, is modified for unit-specific characteristics, in particular familiarity F_i and complexity C_i.

In addition, $T_i(j)$ is modified for reporting and travel time. Since ABC's activities are worldwide spread this correction may have a considerable impact. The corrected time $T'_i(j)$ to perform audit level j in unit i is then given by :

$$T'_i(j) = 1.2 \times T_i(j) + D_i \times 16 \times A(j),$$

where the last term is a correction for travel time, both unit as audit level specific, and the factor 1.2 reflects a typical 20 % extra time for reporting. $A(j)$ equals 2 for j equal to 1 or 2, and 3 for j equal to 3.

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Exhibit 1 : Computation of the Risk Index

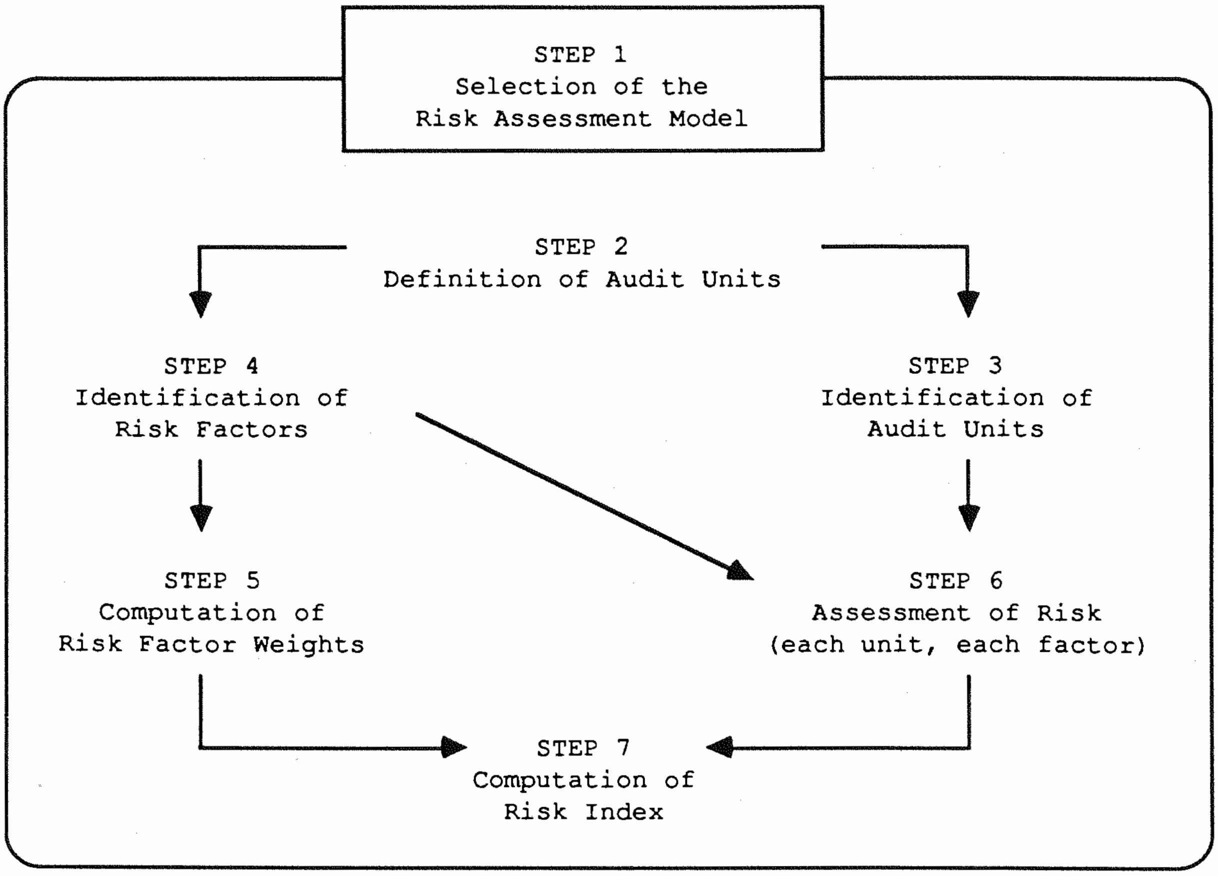

Exhibit 2 : Risk Factors for ABC Inc.

(1) Size
Monetary importance and number of employees.

(2) Internal Control
The quality of the unit's internal control structure, based on previous audit results and the commitment of the unit's management to maintain proper internal controls.

(3) Change
Changes in management and key personnel, business activity, organization, business systems and external environment.

(4) Environment
Political risk, local business practices, and local legal, economical and cultural environment.

(5) Internal and External Pressure
Internal pressure on management such as budgets and procedures and external pressure on management resulting from the operational performance and the general economic condition of the unit.

(6) Type and Scope of Activities
Activities of the unit such as existence of production facilities, R&D, export, and the diversification of the product line.

Exhibit 3 : Nine-point Comparison Scale for Risk Factors

Score	Definition	Explanation
1	Equally important	The two factors are equally important.
3	Slightly more important	Experience and judgement suggest that one factor is slightly more important than the other.
5	Strongly more important	Experience and judgement suggest that one factor is strongly more important than the other.
7	Demonstrabely more important	One factor is clearly more important, and this importance has been demonstrated.
9	Absolutely more important	The evidence showing one factor to be clearly more important is of the highest order of affirmation.

2,4,6,8 are intermediate values when compromise is needed.

Exhibit 4 : Computation of Risk Factor Weights for ABC Inc.

Expert	Comparison Matrix						Weights	
1	1.000	0.200	0.333	1.000	1.000	0.333	.064	L=6.231
	5.000	1.000	3.000	5.000	5.000	3.000	.410	M=0.043
	3.000	0.333	1.000	5.000	5.000	3.000	.261	T=43 (n=14)
	1.000	0.200	0.200	1.000	1.000	0.333	.059	
	1.000	0.200	0.200	1.000	1.000	0.333	.059	
	3.000	0.333	0.333	3.000	3.000	1.000	.148	
2	1.000	0.143	0.111	0.200	0.200	0.333	.028	L=6.298
	7.000	1.000	0.333	3.000	3.000	5.000	.244	M=0.060
	9.000	3.000	1.000	5.000	5.000	5.000	.449	T=39 (n=14)
	5.000	0.333	0.200	1.000	1.000	3.000	.111	
	5.000	0.333	0.200	1.000	1.000	3.000	.111	
	3.000	0.200	0.200	0.333	0.333	1.000	.056	
3	1.000	5.000	0.333	3.000	3.000	0.200	.162	L=6.551
	0.200	1.000	0.333	3.000	3.000	0.333	.093	M=0.110
	3.000	3.000	1.000	9.000	5.000	1.000	.304	T=35 (n=14)
	0.333	0.333	0.111	1.000	0.333	0.111	.032	
	0.333	0.333	0.200	3.000	1.000	0.200	.056	
	5.000	3.000	1.000	9.000	5.000	1.000	.353	
4	1.000	0.143	0.200	0.333	1.000	1.000	.056	L=6.791
	7.000	1.000	5.000	5.000	5.000	5.000	.474	M=0.158
	5.000	0.200	1.000	5.000	3.000	3.000	.218	T=37 (n=14)
	3.000	0.200	0.200	1.000	0.333	0.333	.064	
	1.000	0.200	0.333	3.000	1.000	3.000	.112	
	1.000	0.200	0.333	3.000	0.333	1.000	.078	
5	1.000	5.000	3.000	1.000	3.000	5.000	.370	L=7.587
	0.200	1.000	5.000	5.000	5.000	3.000	.259	M=0.317
	0.333	0.200	1.000	1.000	3.000	3.000	.115	T=32 (n=15)
	1.000	0.200	1.000	1.000	1.000	0.333	.095	
	0.333	0.200	0.333	1.000	1.000	0.200	.050	
	0.200	0.333	0.333	3.000	5.000	1.000	.111	

Exhibit 5 : Pairwise Correlation of Risk Factor Weights for ABC Inc.

	Expert1	Expert2	Expert3	Expert4	Expert5
Expert1	1				
Expert2	.657	1			
Expert3	.192	.252	1		
Expert4	.948	.568	-.109	1	
Expert5	.207	-.201	-.032	.220	1

Exhibit 6 : Consensus Weights for ABC Inc., Method 1

Central Tendency Comparison Matrix						Consensus Weights	
1.000	0.200	0.333	1.000	1.000	0.333	0.064	L=6.231
5.000	1.000	3.000	5.000	5.000	3.000	0.410	M=0.043
3.000	0.333	1.000	5.000	5.000	3.000	0.261	T=43 (n=14)
1.000	0.200	0.200	1.000	1.000	0.333	0.059	
1.000	0.200	0.200	1.000	1.000	0.333	0.059	
3.000	0.333	0.333	3.000	3.000	1.000	0.148	

Exhibit 7 : Consensus Weights for ABC Inc., Method 2

Initial estimate

Z0 = (1/N), i=1..N

Iteration procedure

Step 1 :

V1 =	1.0000	1.0000	1.0000	1.0000	1.0000	
Z1 =	0.1361	0.2958	0.2691	0.0721	0.0777	0.1492
S1 =	0.2935					

Step 2 :

V2 =	1.0897	1.0470	0.8994	1.0904	0.8735	
Z2 =	0.1259	0.3056	0.2730	0.0723	0.0794	0.1439
S2 =	0.2829					

Step 3 :

V3 =	1.0864	1.0448	0.8763	1.0918	0.8512	
Z3 =	0.1247	0.3068	0.2734	0.0724	0.0796	0.1430
S3 =	0.2827					

Step 4 :

V4 =	1.0858	1.0445	0.8733	1.0918	0.8485	
Z4 =	0.1245	0.3070	0.2735	0.0724	0.0797	0.1429
S4 =	0.2827					

Step 5 :

V5 =	1.0858	1.0444	0.8729	1.0919	0.8482	
Z5 =	0.1245	0.3070	0.2735	0.0724	0.0797	0.1429
S5 =	0.2827 minimized					

Final estimate

V = (1.0858, 1.0444, 0.8729, 1.0919, 0.8482)
Z = (0.1245, 0.3070, 0.2735, 0.0724, 0.0797, 0.1429)

Exhibit 8 : Risk Index for ABC Inc.

AUDIT UNIT	SIZE 6%	IC/AH 41%	CHANGE 26%	ENVIR 6%	I/E PRES 6%	T/S ACT 15%	TOTAL RISK
			RISK FACTORS				
A15	4	5	4	4	2	5	4.44
A6	5	5	4	2	2	5	4.38
A20	4	5	4	4	2	2	3.99
A12	4	5	3	4	2	3	3.88
A3	3	5	4	3	2	2	3.87
A21	3	5	3	2	2	4	3.85
A1	3	5	4	2	2	2	3.81
A14	3	4	4	3	2	4	3.76
A24	2	5	4	3	2	1	3.66
A22	2	5	3	1	2	2	3.43
C3	1	4	4	4	2	2	3.40
A9	1	5	3	4	1	1	3.34
C1	1	4	4	4	2	1	3.25
A19	4	3	4	2	2	3	3.20
A8	2	4	3	3	2	2	3.14
A13	4	2	5	3	2	3	3.11
A10	2	4	3	4	2	1	3.05
A26	4	3	3	1	2	4	3.03
B9	2	3	3	3	2	4	3.03
A27	5	1	5	2	2	5	3.00
A4	5	1	5	2	1	5	2.94
A17	2	3	3	4	2	1	2.64
B2	3	2	2	5	2	4	2.54
B3	1	3	2	5	2	2	2.53
B11	1	3	2	5	2	2	2.53
D6	1	3	3	3	2	1	2.52
D7	1	3	3	3	2	1	2.52
D13	1	3	2	4	2	2	2.47
B5	1	3	2	4	2	2	2.47
B8	1	3	2	4	2	2	2.47
A18	3	2	3	2	2	3	2.47
D9	1	3	2	4	2	2	2.47
B7	1	3	3	2	2	1	2.46
C2	1	3	3	2	2	1	2.46
B12	1	3	3	2	2	1	2.46
B4	2	3	2	5	2	1	2.44
E7	1	3	2	3	2	2	2.41
D3	1	3	2	3	2	2	2.41
B10	1	3	2	3	2	2	2.41
D12	1	3	2	3	2	2	2.41
D8	1	3	2	3	2	2	2.41
B1	3	2	2	5	2	3	2.39
D5	1	3	2	5	2	1	2.38
D10	1	3	2	5	2	1	2.38
A16	3	2	3	1	1	3	2.35
A11	4	1	4	1	2	3	2.32
D4	1	3	2	4	2	1	2.32
D2	1	3	2	4	2	1	2.32
D14	1	3	2	3	2	1	2.26
D1	1	3	2	3	2	1	2.26
D11	1	3	2	3	2	1	2.26
A25	2	2	3	2	2	2	2.26
A7	5	1	2	3	3	4	2.19
A23	2	2	2	2	2	3	2.15
B6	2	2	2	3	2	2	2.06
A5	2	1	2	1	3	4	1.89
A2	2	2	1	2	1	2	1.68

Exhibit 9 : Risk Reduction and Time Requirements for ABC Inc.

AUDIT						LEVEL 1		LEVEL 2		LEVEL 3	
UNIT	TRi	MRi	Fi	Ci	Di	RRi(1)	Ti(1)	RRi(2)	Ti(2)	RRi(3)	Ti(3)
A15	4.44	1.00	3	5	1.0	1.24	170	2.17	378	3.10	912
A6	4.38	1.00	4	5	0.5	1.28	143	2.25	333	3.21	816
A20	3.99	1.00	3	2	1.0	1.08	101	1.88	205	2.69	480
A12	3.88	1.00	3	3	0.5	1.04	108	1.81	246	2.59	600
A3	3.87	1.00	4	2	1.0	1.09	095	1.91	190	2.73	444
A21	3.85	1.00	3	4	0.5	1.03	131	1.80	304	2.57	744
A1	3.81	1.00	3	2	1.5	1.01	117	1.77	221	2.53	504
A14	3.76	1.00	3	4	1.0	0.99	147	1.74	320	2.48	768
A24	3.66	1.00	3	1	1.0	0.96	078	1.68	147	2.39	336
A22	3.43	1.00	4	2	0.5	0.92	079	1.62	174	2.31	420
C3	3.40	1.00	3	2	1.0	0.86	101	1.51	205	2.16	480
A9	3.34	1.00	3	1	1.0	0.84	078	1.47	147	2.11	336
C1	3.25	1.00	3	1	0.5	0.81	062	1.42	131	2.03	312
A19	3.20	1.00	3	3	1.0	0.79	124	1.39	262	1.98	624
A8	3.14	1.00	4	2	0.5	0.81	079	1.42	174	2.03	420
A13	3.11	1.00	2	3	1.0	0.72	132	1.26	282	1.79	672
A10	3.05	1.00	3	1	1.0	0.74	078	1.29	147	1.85	336
A26	3.03	1.00	4	4	0.5	0.77	122	1.35	280	1.93	684
B9	3.03	1.00	1	4	1.0	0.65	166	1.14	368	1.62	888
A27	3.00	1.00	5	5	1.0	0.80	147	1.40	320	2.00	768
A4	2.94	1.00	5	5	0.0	0.78	115	1.36	288	1.94	720
A17	2.64	1.00	3	1	1.0	0.59	078	1.03	147	1.48	336
B2	2.54	1.00	2	4	1.0	0.52	157	0.92	344	1.31	828
B3	2.53	1.00	2	2	1.0	0.52	107	0.91	219	1.30	516
B11	2.53	1.00	2	2	1.0	0.52	107	0.91	219	1.30	516
D6	2.52	1.00	1	1	0.5	0.49	070	0.85	150	1.22	360
D7	2.52	1.00	1	1	0.5	0.49	070	0.85	150	1.22	360
D13	2.47	1.00	2	2	1.0	0.50	107	0.87	219	1.25	516
B5	2.47	1.00	2	2	1.0	0.50	107	0.87	219	1.25	516
B8	2.47	1.00	1	2	1.0	0.47	113	0.82	234	1.18	552
A18	2.47	1.00	3	3	0.5	0.53	108	0.93	246	1.32	600
D9	2.47	1.00	2	2	1.0	0.50	107	0.87	219	1.25	516
B7	2.46	1.00	3	1	1.0	0.53	078	0.92	147	1.31	336
C2	2.46	1.00	3	1	1.0	0.53	078	0.92	147	1.31	336
B12	2.46	1.00	3	1	1.0	0.53	078	0.92	147	1.31	336
B4	2.44	1.00	2	1	1.0	0.49	082	0.86	157	1.22	360
E7	2.41	1.00	1	2	0.5	0.45	097	0.79	218	1.13	528
D3	2.41	1.00	1	2	1.0	0.45	113	0.79	234	1.13	552
B10	2.41	1.00	1	2	1.0	0.45	113	0.79	234	1.13	552
D12	2.41	1.00	1	2	1.0	0.45	113	0.79	234	1.13	552
D8	2.41	1.00	1	2	1.0	0.45	113	0.79	234	1.13	552
B1	2.39	1.00	2	3	1.0	0.47	132	0.83	282	1.18	672
D5	2.38	1.00	2	1	1.0	0.47	082	0.82	157	1.17	360
D10	2.38	1.00	1	1	1.0	0.44	086	0.77	166	1.10	384
A16	2.35	1.00	5	3	0.5	0.54	093	0.95	208	1.35	504
A11	2.32	1.00	4	3	0.5	0.50	100	0.88	227	1.25	552
D4	2.32	1.00	1	1	1.0	0.42	086	0.74	166	1.06	384
D2	2.32	1.00	1	1	0.5	0.42	070	0.74	150	1.06	360
D14	2.26	1.00	1	1	1.0	0.40	086	0.71	166	1.01	384
D1	2.26	1.00	1	1	0.5	0.40	070	0.71	150	1.01	360
D11	2.26	1.00	1	1	0.5	0.40	070	0.71	150	1.01	360
A25	2.26	1.00	3	2	1.0	0.45	101	0.79	205	1.13	480
A7	2.19	1.00	5	4	0.5	0.48	112	0.83	256	1.19	624
A23	2.15	1.00	4	3	0.5	0.44	100	0.76	227	1.09	552
B6	2.06	1.00	2	2	1.0	0.36	107	0.63	219	0.90	516
A5	1.89	1.00	4	4	0.5	0.34	122	0.59	280	0.85	684
A2	1.68	1.00	4	2	0.5	0.26	079	0.45	174	0.65	420

Exhibit 10 : Audit Resource Allocation for ABC Inc.

AUDIT UNIT	AUDIT LEVEL	RISK REDUCTION
A15	2	2.17
A6	2	2.25
A20	2	1.88
A12	2	1.81
A3	2	1.91
A21	1	1.03
A1	2	1.77
A14	1	0.99
A24	2	1.68
A22	2	1.62
C3	2	1.51
A9	2	1.47
C1	2	1.42
A19	1	0.79
A8	2	1.42
A13	1	0.72
A10	2	1.29
A26	1	0.77
B9	0	0.00
A27	1	0.80
A4	1	0.78
A17	2	1.03
B2	0	0.00
B3	1	0.52
B11	1	0.52
D6	2	0.85
D7	2	0.85
D13	1	0.50
B5	1	0.50
B8	0	0.00
A18	1	0.53
D9	1	0.50
B7	2	0.92
C2	2	0.92
B12	2	0.92
B4	2	0.86
E7	1	0.45
D3	0	0.00
B10	0	0.00
D12	0	0.00
D8	0	0.00
B1	0	0.00
D5	2	0.82
D10	2	0.77
A16	1	0.54
A11	1	0.50
D4	1	0.42
D2	1	0.42
D14	1	0.40
D1	1	0.40
D11	1	0.40
A25	1	0.45
A7	0	0.00
A23	0	0.00
B6	0	0.00
A5	0	0.00
A2	0	0.00
Total risk reduction		43.09